33/54)

$$F(x) = x f(x) \quad \infty \quad F'''(x)$$

$$F'(x) = (1)(f(x)) + (x)(f'(x))$$

$$= f(x) + x f'(x)$$

$$F''(x) = f'(x) + (1)(f'(x)) + (x)(f''(x))$$

$$= 2f'(x) + x f''(x)$$

$$F'''(x) = 2f''(x) + (1)(f''(x)) + (x)(f'''$$

$$F'''(x) = 3f''(x) + x f'''(x)$$

b) $F(x) = x f(x)$

guess what

$$F^{(n)}(x) = n f^{(n-1)}(x) + x f^{(n)}(x$$

$(n>2)$

3.5/43)

$y = x\cos(3x)$; $x = \pi$

Find tangent line to $y = x\cos(3x)$ at $x = \pi$

Finding the equation of a tangent line is a specific instance of finding the equation of a LINE

SLOPE

derivative! in particular... the VALUE of the derivative fⁿ at the pt

a point

$x = \pi$

$y = ?$

equation of LINE

equation of tangent LINE

$y = x\cos(3x)$

$y' = (1)(\cos(3x)) + (x)(-\sin(3x)\,3)$

$y'\big|_{x=\pi} = \cos(3\pi) - 3\pi\sin(3\pi)$

$= \cos(2\pi + \pi) - 3\pi\sin(2\pi + \pi)$

$= -1 - 3\pi(0)$

$m = -1$

$y = x\cos(3x)$

$y\big|_{x=\pi} = \pi\cos(3\pi)$

$= \pi(-1) = -\pi$

$m = -1$

$Pt = (\pi, -\pi)$

eqⁿ:

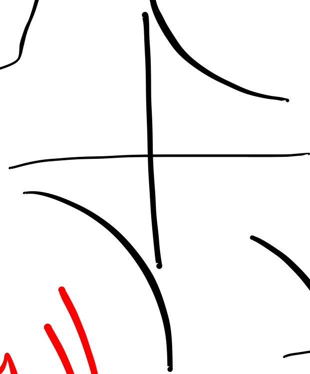
$y - (-\pi) = (-1)(x - (\pi))$

3.6/21)

$3x^2-4y^2=7$ find $\frac{d^2y}{dx^2}$ by ID (imp diff)

$ax^2+by^2=r^2$ $b<0$

$\frac{dy}{dx}=\frac{3x}{4y}$

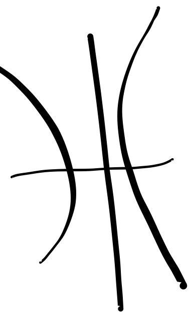


deriv 1

$$6x-8y\frac{dy}{dx}=0$$

deriv 2

$$6-8\left(\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)+(y)\left(\frac{d^2y}{dx^2}\right)\right)=0$$



Solve for $\frac{d^2y}{dx^2}$ . . . . .

$$y\frac{d^2y}{dx^2}=8\left(\frac{dy}{dx}\right)^2-6$$

$$\frac{d^2y}{dx^2}=\frac{8\left(\frac{3x}{4y}\right)^2-6}{y}$$

$$\frac{dy}{dx} = \frac{3x}{4y}$$

$$\frac{d^2y}{dx^2} = \frac{(3)(4y) - (3x)\left(4\frac{dy}{dx}\right)}{(4y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{12y - 12x\left(\frac{3x}{4y}\right)}{(4y)^2}$$

chain rule

for everything else

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

We know

Power $\frac{d}{dx} x^n = n x^{n-1}$

Product $\frac{d}{dx}(f \cdot g) = f'g + fg'$

Quotient $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{(g)^2}$

$$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \cdot f'(x)$$

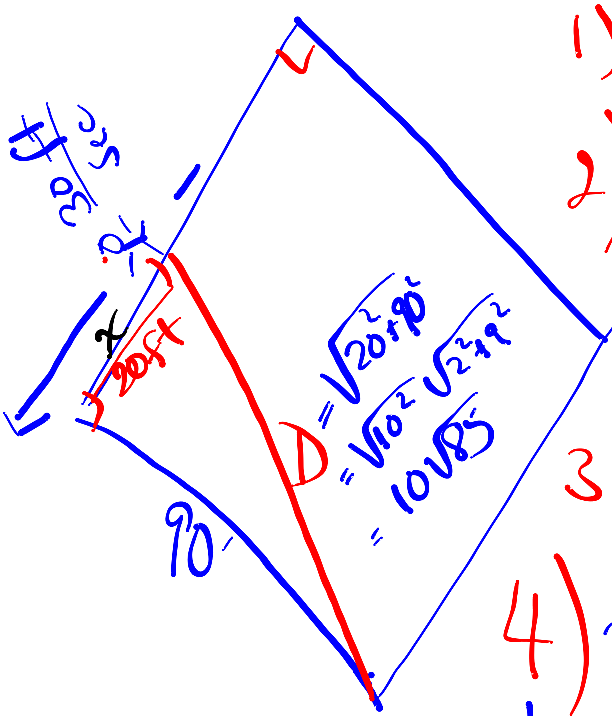
$$\frac{d}{dx}\left[\sin(x^2+1)\right] \cdot \left[\cos x\right]^n =$$

$$\frac{d}{dx}[\cos x]^n$$

$$\left(\frac{d}{dx}(\sin(x^2+1))\right)(\cos^n x) + (\sin(x^2+1))\left(\frac{d}{dx}\cos^n x\right)$$

$$\left(\cos(x^2+1)\cdot(2x)\right)(\cos^n x) + \left(\sin(x^2+1)\right)\left(n[\cos x]^{n-1}(-\sin x)\right)$$

3.7)



$D = \sqrt{20^2+90^2} = \sqrt{10^2(2^2+9^2)} = 10\sqrt{85}$

1) picture

2) $x^2 + 90^2 = D^2$

$\frac{d}{dt} \ldots$

3) $2x\frac{dx}{dt} + 0 = 2D\frac{dD}{dt}$

4) $2(20)(-30) = 2(10\sqrt{85})\frac{dD}{dt}$

$$\frac{dD}{dt} = \frac{-1200}{20\sqrt{85}} \ \frac{ft}{sec}$$

3.7/1 $y=3x+5$

a) given $\frac{dx}{dt}=2$, find $\frac{dy}{dt}$ when $x=1$

step 2 $y=3x+5$

step 3 $\frac{dy}{dt}=3\frac{dx}{dt}$

step 4 $\frac{dy}{dt}=3(2)=6$

b) given $\frac{dy}{dt}=-1$, find $\frac{dx}{dt}$ when $x=0$

$y=3x+5$

$\frac{dy}{dt}=3\frac{dx}{dt}$

$(-1)=3\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{-1}{3}$

3.7/13 $\quad x^2+y^2=1$

steps 2

S3 $\quad 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

(a) $\frac{dx}{dt}=1$; $(x,y)=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$$2\left(\frac{1}{2}\right)(1)+2\left(\frac{\sqrt{3}}{2}\right)\frac{dy}{dt}=0$$

Solve 4 $\frac{dy}{dt}$ $\quad \frac{dy}{dt}=-\frac{1}{\sqrt{3}}$ aka $-\frac{\sqrt{3}}{3}$

b) $\frac{dy}{dt}=-2$; $(x,y)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$$2\left(\frac{\sqrt{2}}{2}\right)\frac{dx}{dt}+2\left(\frac{\sqrt{2}}{2}\right)(-2)=0$$

Solve for $\frac{dx}{dt}$ $\quad \frac{dx}{dt}=\frac{2\sqrt{2}}{\sqrt{2}}=2$

Format

one side ONLY

organized work

"one column" only

Use paper - it will NOT be wasted

3.6/23 $x^3y^3-4=0$ find $\frac{d^2y}{dx^2}$

$$(3x^2)(y^3)+(x^3)\left(3y^2\frac{dy}{dx}\right)=0$$

$$\frac{dy}{dx}=\frac{-3x^2y^3}{3x^3y^2}=-\frac{y}{x}$$

second

$$\frac{d^2y}{dx^2}=-\frac{\left(\frac{dy}{dx}\right)(x)-(y)(1)}{(x)^2}$$

$$\frac{d^2y}{dx^2}=-\frac{\left(-\frac{y}{x}\right)(x)-y}{x^2}$$

$$\frac{d^2y}{dx^2}=\frac{2y}{x^2}$$

$$3x^2y^3 + 3x^3y^2\frac{dy}{dx} = 0$$

$$x^2y^3 + x^3y^2\frac{dy}{dx} = 0$$

$$\left[(2x)(y^3) + (x^2)\left(3y^2\frac{dy}{dx}\right)\right] +$$

$$(3x^2)\left(y^2\frac{dy}{dx}\right) + \left(2y\frac{dy}{dx}\right)\left(x^3\frac{dy}{dx}\right) + (x^3y^2)\left(\frac{d^2y}{dx^2}\right) = 0$$

$$4x^2y^2\frac{dy}{dx} + 2xy^3 + 2x^3y\left(\frac{dy}{dx}\right)^2 + x^3y^2\left(\frac{d^2y}{dx^2}\right) = 0$$

$$4x^2y^2\left(\frac{-y}{x}\right) + 2xy^3 + 2x^3y\left(\frac{-y}{x}\right)^2 + x^3y^2\frac{d^2y}{dx^2} = 0$$

$$-4xy^3 + 2xy^3 + 2xy^3 + x^3y^2\frac{d^2y}{dx^2} = 0$$

$$\frac{d}{dx}(f(g\cdot h)) = f'gh + f(gh)'$$

$$= f'gh + f(g'h + gh')$$

$$= f'gh + fg'h + fgh'$$

3.5/51

find y''

$$x\cos(5x) - \sin^2 x = y$$

$$(1)(\cos(5x)) + (x)(-5\sin(5x)) - 2\sin x\cos x = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \cos(5x) - 5x\sin(5x) - 2\sin x\cos x$$

$$\frac{d^2y}{dx^2} = -5\sin(5x) - 5\Big((1)(\sin(5x)) + (x)(5\cos(5x))\Big)$$
$$-2\Big((\cos x)(\cos x) + (\sin x)(-\sin x)\Big)$$

$$\frac{d^2y}{dx^2} = -5\sin(5x) - 5\sin(5x) - 25x\sin(5x)\cos(5x)$$
$$-2\cos^2 x + 2\sin^2 x$$

3.3/51 $y = x^3 + 3x + 1$ satisfies

$y''' + xy'' - 2y' = 0$ → differential equation

Segue, show that $x = 2$ satisfies $x^2 - 4 = 0$

$(2)^2 - 4 = 0$

$4 - 4 = 0$

$0 = 0$ ✓

$y = x^3 + 3x + 1$

$y' = 3x^2 + 3$

$y'' = 6x$

$y''' = 6$

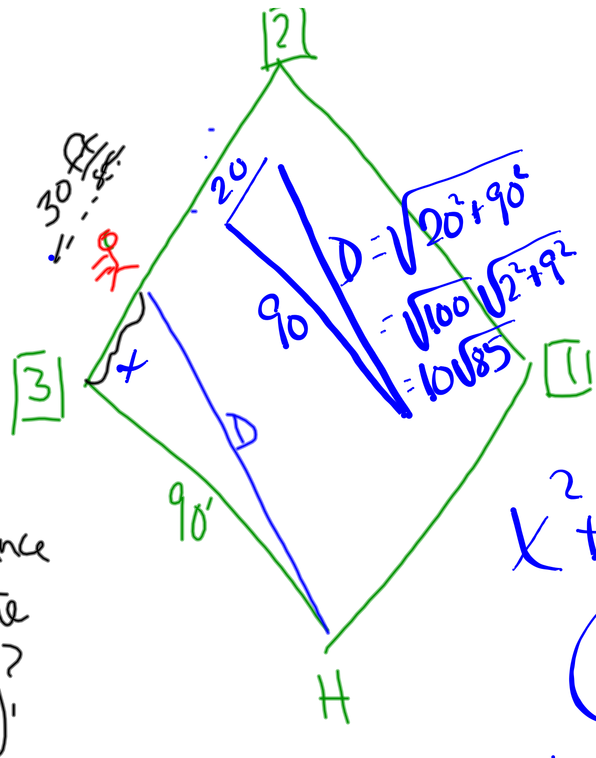
$y''' + xy'' - 2y' = 0$

$(6) + x(6x) - 2(3x^2 + 3)$

$= 6 + 6x^2 - 6x^2 - 6 = 0$

3.7

Q: When the runner is 20' from 3rd base, how is the distance from home plate changing?



$$D = \sqrt{20^2 + 90^2} = \sqrt{100}\sqrt{2^2+9^2} = 10\sqrt{85}$$

Step 1

picture

Step 2

EQ always true

$$x^2 + 90^2 = D^2$$

Step 3

$$\frac{d}{dx}(x(t))^2$$

$$2(x(t))\frac{d}{dt}x(t)$$

$$2x\frac{dx}{dt} + 0 = 2D\left(\frac{dD}{dt}\right)$$

Step 4

substitute in stuff

$$2(20)(-30) = 2(10\sqrt{85})\frac{dD}{dt}$$

$$\frac{-1200}{20\sqrt{85}} = \frac{dD}{dt}$$

$$-\frac{12}{17}\sqrt{85}$$

$$\parallel$$

$$\frac{-60\sqrt{85}}{85} = \frac{-60}{\sqrt{85}}\ ft/sec$$

2.5 ⇒ Continuity

→ 3 conditions

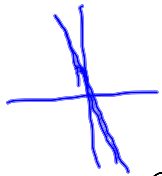
2.6 ⇒ limits of trig fn

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

3.1 avg rate of chg vs.

instantaneous rate of chg

3.7/2)

$Eq^n: x+4y=3$ 

a) Given that $\frac{dx}{dt}=1$, find $\frac{dy}{dt}$ when $x=2$

Eqn stp 2 $\quad x+4y=3$

$\frac{d}{dt}$ stp3 $\quad \frac{dx}{dt}+4\frac{dy}{dt}=0$

sub stp4 $\quad (1)+4\frac{dy}{dt}=0$

$\therefore \frac{dy}{dt}=\frac{-1}{4}$ ✓

b) given that $\frac{dy}{dt}=4$; find $\frac{dx}{dt}$ when $x=3$

stp 2) $\quad x+4y=3 \qquad \frac{d}{dt}\left[4(y(t))\right]$

stp3) $\quad \frac{dx}{dt}+4\frac{dy}{dt}=0 \qquad 4\frac{d}{dt}\left[y(t)\right]$

stp 4) $\quad \frac{dx}{dt}+4(4)=0 \qquad 4\frac{dy}{dt}$

$\therefore \frac{dx}{dt}=-16$

~~Imp Diff~~

$y=f(x)$

$y'=\frac{dy}{dx}=f'(x)$